Name:
Welcome to AP Precalculus!

Instructions for completing the AP Precalculus summer assignment:

- Answer all questions to the best of your ability. If you do not know how to complete a question on the assignment, use your resources (i.e. internet, friends, etc.) to try to learn the concept required. However...
- Maintain integrity in completing the assignment. Cheating is prohibited. This assignment is meant to gauge your understanding of specific topics in mathematics prior to the start of the course. Cheating on the assignment may communicate to me that you understand a topic that you truly do not.
- Try your best. I expect that full effort is put into completing the assignment and that all work is shown. This assignment will count as your first grade for Quarter 1.

I look forward to getting to know you all in September. We have many interesting topics in this course ahead of us and I am excited to share this experience with all of you. If you have any questions or concerns over the summer, feel free to send me an email.

Mr. O'Brien
zobrien@newbedfordschools.org

## Summer Assignment AP Precalculus 2023

Finding the Slope of a Line Algebraically:
How do we find the slope (denoted as $m$ ) of a line given two points $\left(x_{1}, y_{1}\right)$ and $\left(x_{2}, y_{2}\right)$ ? $m=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}$

Example: Find the slope of the line containing the points $(-6,-1)$ and $(3,2)$.

$$
m=\frac{2-(-1)}{3-(-6)}=\frac{3}{9}=\frac{1}{3}
$$

Find the slope for the line that contains each pair of points.
a) $(2,3)$ and $(4,15)$
b) $(3,6)$ and $(5,4)$
c) $(-2,-5)$ and $(3,7)$
d) $\left(-\frac{1}{2}, 0\right)$ and $\left(2, \frac{3}{2}\right)$
e) $(2,-4)$ and $(-1,5)$
f) $\left(\frac{1}{4}, 2\right)$ and $\left(\frac{3}{4}, \frac{5}{2}\right)$

## Summer Assignment AP Precalculus 2023

Writing an Equation of a Line Using Point-Slope Form:
What is point-slope form?
$y-y_{1}=m\left(x-x_{1}\right)$
Example: Find the equation of the line containing the points $(-6,-1)$ and $(3,2)$ using point-slope form.

Solution: First find the slope between the two points.
$m=\frac{2-(-1)}{3-(-6)}=\frac{3}{9}=\frac{1}{3}$
Now, write the equation of the line.
$y-(-1)=\frac{1}{3}(x-(-6))$
$y+1=\frac{1}{3}(x+6)$

Write an equation for each line in point-slope form.
a) containing the point $(4,2)$ and with a slope of $\frac{1}{2}$
b) containing the point $(-1,3)$ and with $m=2$
c) containing the points $(1,1)$ and $(4,5)$.

## Summer Assignment AP Precalculus 2023

## Factoring Polynomials:

Below is the thought process for factoring polynomials!
Step 1: Identify the GCF between the terms and factor it out if the GCF $\neq 1$. If the leading term (term with the highest power) is negative then you must also factor out a negative first! Then look inside the parentheses and see if it can factored further.
Step 2: If the GCF between all the terms is 1 , then look to see how many terms you have!

Technique for 4 terms:
Use group factoring! Technique is explained below.

- Group the first two terms and the last two terms together.
- Then factor out the GCF from each of the two sets of parentheses.
- After you factor out the GCF's, looking at your new expression with two terms, factor out the GCF again which will be a binomial.

Techniques for 3 terms (trinomials): For trinomials of the form $\mathrm{ax}^{2}+b x+c$
$\mathrm{a}=1$ : (technique explained below)
Think of factors of the c-value that add up to b-value
$a \neq 1:($ technique explained below)

- Multiply the a and c value and find factors that somehow add/subtract to the middle term.
- Expand out the expression into 4 terms.
- Use group factoring method to factor the expression.

Techniques for 2 terms (binomials):

## Difference of perfect squares

$a^{2}-b^{2}=(a-b)(a+b)$

## Sum/Difference of perfect cubes

$$
\begin{aligned}
& a^{3}+b^{3}=(a+b)\left(a^{2}-a b+b^{2}\right) \\
& a^{3}-b^{3}=(a-b)\left(a^{2}+a b+b^{2}\right)
\end{aligned}
$$

## Summer Assignment AP Precalculus 2023

Factor the polynomials below completely!
a) $2 x^{2}+12 x+16$
b) $x^{2}-3 x+10$
c) $x^{2}+2 x-24$
d) $x^{2}-16$
e) $8 x^{3}+27$
f) $2 x^{3}-18 x$
g) $5 x^{2}-3 x-2$
h) $3 x^{2}+7 x+2$

## Summer Assignment AP Precalculus 2023

Zero Product Property: This property states that if $a b=0$ then either $a=0$ or $b=0$ (or both are zero).

Using the Zero Product Property to solve equations:
Example: Solve $(3 x+1)(x-2)=0$ for $x$.
$3 x+1=0$ or $x-2=0$
$3 x=-1$ or $x=2$
$x=-\frac{1}{3}$ or $x=2$
Solve the equations below for $x$.
a) $2 x(x-1)=0$
b) $(2 x+5)(x-4)=0$
c) $x(3 x+4)(2 x-1)=0$
d) $(x-3)\left(x+\frac{1}{2}\right)=0$
e) $\left(\frac{3}{2} x+4\right)\left(2 x-\frac{1}{3}\right)=0$
f) $2(x-10)(4 x+5)=0$

## Summer Assignment AP Precalculus 2023

Solving Quadratic Equations by Factoring:
Example 1: Solve $15 x^{2}+5 x=0$ for $x$.
Factor the left hand side of the equation and use the zero product property to solve for $x$.
$5 x(3 x+1)=0$
$5 x=0$ or $3 x+1=0$
$x=0$ or $x=-\frac{1}{3}$
Example 2: Solve $x^{2}+3 x-14=-2 x+10$ for $x$.
Bring all the terms to one side of the equation. Then, factor the left hand side of the equation and use the zero product property to solve for $x$.
$x^{2}+5 x-24=0$
$(x+8)(x-3)=0$
$x+8=0$ or $x-3=0$
$x=-8$ or $x=3$

Solve the equations below for $x$ by factoring.
a) $12 x^{2}+8 x=0$
b) $x^{2}+5 x+4=0$
c) $3 x^{2}+11 x-6=-2$
d) $4 x^{2}+3 x-11=3 x-2$

## Summer Assignment AP Precalculus 2023

Exponent Rules:

- $x^{m} \cdot x^{n}=x^{m+n}$
Ex: $2 x^{2} \cdot 3 x^{5}=6 x^{2+5}=6 x^{7}$
- $\frac{x^{m}}{x^{n}}=x^{m-n}$
Ex: $\frac{8 x^{9}}{2 x^{5}}=4 x^{9-5}=4 x^{4}$
- $\left(x^{m}\right)^{n}=x^{m n}$
Ex: $\left(2 x^{3}\right)^{2}=2^{2} x^{3 \cdot 2}=4 x^{6}$
- $x^{0}=1$ where $x \neq 0$
Ex: $(9 x y z)^{0}=1$
- $x^{-n}=\frac{1}{x^{n}}$
Ex: $x^{-3}=\frac{1}{x^{3}}$
- $\sqrt[n]{x^{m}}=x^{\frac{m}{n}}$
Ex: $\sqrt[4]{x^{3}}=x^{\frac{3}{4}}$

Remember that for the square root symbol the root number is 2 but it is never labeled.

Simplify the expressions and make sure to use positive exponents in your final answer.
a) $7 x^{6} \cdot 2 x^{4}$
b) $\frac{6 x^{7} y^{8}}{2 x^{3} y}$
c) $-2(8 x y)^{0}$
d) $\left(4 x^{4} y^{-2}\right)^{3}$
e) $\left(3 a b^{6}\right)^{2}\left(-2 a^{2} b^{-2}\right)^{3}$
f) $\sqrt{\frac{x^{2}}{y^{2}}}$

## Summer Assignment AP Precalculus 2023

Rewrite the following expressions using rational exponents.
Example: $\frac{1}{\sqrt[3]{x^{2}}}=\frac{1}{x^{\frac{2}{3}}}=x^{-\frac{2}{3}}$
a) $\sqrt[5]{x^{3}}+\sqrt{x}$
b) $\sqrt[3]{x+1}$
c) $\frac{1}{\sqrt[3]{x^{5}}}$
d) $\frac{1}{x^{10}}-\frac{3}{x}$
e) $\frac{1}{2 x^{6}}+\frac{1}{4 \sqrt{x}}$
f) $2 \sqrt{x^{7}+4}$

Rewrite the following expressions using roots/positive exponents.
Example: $x^{-\frac{2}{3}}+2 x^{-7}=\frac{1}{x^{\frac{2}{3}}}+\frac{2}{x^{7}}=\frac{1}{\sqrt[3]{x^{2}}}+\frac{2}{x^{7}}$
a) $x^{-\frac{5}{2}}+x^{\frac{1}{4}}$
b) $x^{-8}+6 x^{-3}$
c) $(2 x+1)^{-16}$
d) $\frac{3}{2} x^{-1}$
e) $\left(3 x^{5}+10\right)^{-\frac{1}{2}}$
f) $\frac{1}{(3 x)^{\frac{4}{5}}}$

## Summer Assignment AP Precalculus 2023

Multiplying Binomials using FOIL Method:
FOIL stands for First Outer Inner Last!
Example: Multiply $(2 x+1)(x+4)$
Solution: $(2 x+1)(x+4)=2 x^{2}+8 x+x+4=2 x^{2}+9 x+4$

Multiply the binomials below and please show work!
a) $(x+2)(x-3)$
b) $(3 x-2)(2 x+7)$
c) $(7 x-4)(x-3)$
d) $(x+5)^{2}$
e) $(3 x-2)^{2}$
f) $(x+9)(x-9)$


Use the unit circle above to fill in the tables below! Recall that the coordinates on the unit circle are $(\cos (\theta), \sin (\theta))$.

| $\theta$ | $\cos (\theta)$ | $\sin (\theta)$ |
| :--- | :--- | :--- |
| $0 \& 2 \pi$ |  |  |
| $\frac{\pi}{6}$ |  |  |
| $\frac{\pi}{4}$ |  |  |
| $\frac{\pi}{3}$ |  |  |
| $\frac{\pi}{2}$ |  |  |
| $\pi$ |  |  |
| $\frac{3 \pi}{2}$ |  |  |


| $\theta$ | $\cos (\theta)$ | $\sin (\theta)$ |
| :--- | :--- | :--- |
| $\frac{5 \pi}{6}$ |  |  |
| $\frac{3 \pi}{4}$ |  |  |
| $\frac{2 \pi}{3}$ |  |  |
| $\frac{7 \pi}{6}$ |  |  |
| $\frac{5 \pi}{4}$ |  |  |
| $\frac{4 \pi}{3}$ |  |  |

## Summer Assignment AP Precalculus 2023

Trigonometric Functions:
Recall the six trigonometric functions: sine, cosine, tangent, secant, cosecant, cotangent.

Below are some definitions for the trigonometric functions!

$$
\begin{array}{lll}
\tan x=\frac{\sin x}{\cos x} & \sec x=\frac{1}{\cos x} & \csc x=\frac{1}{\sin x} \\
\cot x=\frac{1}{\tan x} & \cot x=\frac{\cos x}{\sin x} &
\end{array}
$$

Evaluate the following using the definitions above and the unit circle!
a) $\tan \left(\frac{\pi}{6}\right)$
b) $\tan (0)$
c) $\tan \left(\frac{\pi}{4}\right)$
d) $\tan \left(\frac{\pi}{3}\right)$
e) $\tan \left(\frac{\pi}{2}\right)$
f) $\sec \left(\frac{\pi}{6}\right)$
g) $\sec (0)$
h) $\sec \left(\frac{\pi}{4}\right)$
i) $\sec \left(\frac{\pi}{3}\right)$
j) $\sec \left(\frac{\pi}{2}\right)$
k) $\csc (0)$
l) $\csc \left(\frac{\pi}{4}\right)$
m) $\csc \left(\frac{\pi}{3}\right)$
n) $\cot \left(\frac{\pi}{2}\right)$
o) $\cot \left(\frac{\pi}{6}\right)$
p) $\cot (0)$


Given the graph above answer the following parts! Make sure you use interval notation.
a) Domain:
b) Range:
c) $x$-intercept(s):
d) $y$-intercept:
e) Increasing:
f) Decreasing:
g) End behavior:

Congratulations! You have completed the AP Precalculus summer assignment :)


